# The Echoes of Bandwagon on Innovation in a Complex Small World Live Network

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**Abstract**

The role of demand on the diffusion of innovation has received invigorated attention recently with the application of network-based models to the behaviour of consumers. New achievements on consumer interaction in a complex environment have highlighted the dynamic of the markets and society as a whole. Small world network (SWN) and word-of-mouth behaviour have been largely used to model the interaction among consumers. Notwithstanding two main aspects of this new literature remains uncovered: i) the interaction between firms and consumers and ii) the source of the network properties based on consumer and firm behaviour. In this article, we build a narrative and an agent-based model to tackle both topics. Allowing for high complex interaction between consumer-consumer and firm-consumer, we offer more explanation about the firm strategy in choosing a process or product innovation and its effects on the pattern of macroeconomic dynamics of the technical change. Additionally, we move forward suggesting an endogenous micro foundation for the network based on the consumer awareness, which we name *small world live network* (SWLN). Among other details, we show that the population of heterogeneous agents and degree of awareness exert a great force on the transition of an economy based on process innovation to product innovation, changing the pace with which an economy develops.

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## 1. Introduction: The Echoes of Bandwagon

Dating back from Schumpeter (1934), literature on innovation has evolved to the point of leading its object of study to the status of one of the main forces driving economic growth and development. The fact that economists now have a much better understanding about Solow's TFP[1] has certainly something to do with understanding how the engine of innovation is greased. In this paper, we suggest that one of the cogwheels of this engine is the bandwagon behaviour of consumers, its impact on innovation diffusion and, ultimately, on the firm's decision to engage on a certain type of innovative process.

A lot has changed since VHS and innovation are now viewed more as a survivability necessity than a profit maximization luxury. New high-tech products such as smartphones, TV's and computers presents a much higher rate of creative destruction. Product innovation does not seem to slow down as predicted by Product Life Cycle (PLC) theory yielding great differentiation that creates changes ranging from small to ones big enough to challenge the dominant design. The fact is that these industries cannot rely on market expansion, already saturated, and even less on repeated purchases of durable goods. The only solution is then to engage heavily on R&D[2] to be able to sustain a constant de-maturity process (see Higuchi and Trout 2008) in which the pattern of innovation diffusion plays a key role in success.

First innovation diffusion models such as Bass (1969) only captured the macroeconomic evolution of innovation diffusion considering consumers as completely homogeneous. But the recent increase in computational capacities and its popularization in economics are key factors to understand why these subjects are back in the spotlight (Kiesling et al. 2012). Demand heterogeneity and social processes such as word of mouth or social learning (Ellison and Fudenberg 1995; Lippert and Spagnolo 2011) suit themselves much better in complex environments and models that cannot generate useful analytical closed solutions other than through simulation. It is also very hard to fit consumer interaction into aggregated equilibrium models which only reinforces the need for proper agent-based models to capture complex emergent phenomena.

We propose to take a step further in innovation theory and investigate how the emergent pattern of diffusion from consumers bandwagon effect shapes the market and influences the firm decision between process and product innovation. This interactive behaviour is determinant to understand how the speed of technological change shape societies and promotes economic development and welfare.

In order to do so, we combine the stochastic threshold idea with consumer profiles to develop a complex agent-based model in a micro-founded and endogenously built graph framework in which the edges are dynamic and the acquaintances are defined by a radius around each node (consumer). Consumers are heterogeneous with respect to what we call *profile* towards new products and follow the distribution proposed by Moore (2005) as a baseline. Since our consumers do not have information about global demand or preferences, the probability to change consumption choices plays the role of stochastic thresholds. Consumers get information about quality and prices by evaluating the product and firms but mainly by interacting with each other when they are inside each other's awareness radius (similar to Chugh and Bazerman 2007). Consumers then evaluate how many others are using a given product or technology

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[1]Total Factor Productivity.

[2]Research and Development.

and decide to change their product of choice or not. Firms, on the other hand, cannot see individual preferences, but analyse market saturation and concentration to decide on the amount of R&D investment and between **process** and **product** innovation. Process innovation increases productivity and therefore decreases price, and product innovation introduces new good which will dynamically diffuse and displace the old one and increases the price.

The paper is organized as follows: section 2 discusses the literature on bandwagon preferences and social networks. Sections and 4 respectively present the social network framework through which our agents interact and a formal agent-based model in which consumers have bandwagon behaviour and influences process and product innovation decision by firms. Section 5 presents and discusses the results and, finally, section 6 brings the conclusions.


## 2. Interdependent Preferences, Bandwagon Behaviour and Diffusion

The effects of interactions between consumers is an ancient issue. Early economist such as Smith (1776, chapter 11), Marshall (1890, book III, chapter ii), Veblen (1899) and others recognize the interdependence of behaviour consumers. These early works share the common view that the preferences among the consumers are not independent. Despite this, the interdependence, or as we refer today, the interaction, remains a challenge even after almost 40 years after the alert of Pollak (1978). The role of demand side in the innovation diffusion process remains a challenge even today, as recognised by many (Metcalfe 2001; Valente 2012; Caliari, Ruiz, and Valente 2017).

One of the first model to investigate the interdependent preference was Gaertner (1974) who inspired Pollak (1976) by using a linear utility function where current preferences depend on the lagged consumption of other consumers. Recent works have formalized such interdependence using richer analytical models by introducing more diversified behaviour and going beyond the endogenous preferences issue. An important contribution was given by Cowan, Cowan, and Swann (1997, 2004) who analyse this interaction in a context where the consumer has three different behaviours regarding the others consumers depending on whether they belong to a "similar", "contrasting" or "aspirational" peer group. A consumer reproduces the behaviour of similar consumers, avoids contrasting one and wishes to move to aspiration group when deciding what to demand. When a member of the aspired group is imitated, he or she tries to move out to new products, stimulating the variety, in order to remain distinguished from the others. A similar (in the spirit, not in the maths) model was developed by Giovinazzo and Naimzada (2015) who interact "snob" and "bandwagoner" consumers generating cycles in the form of a dynamic replicator mechanism. A bandwagoner consumer copy a snob one and a snob consumer searches for differentiating tastes from the bandwagoner generating a perpetual movement of the populational density.

The literature above mentioned has focused on the tastes changing. The phenomenon of social interaction by self and its influence on the patterns of consumption and its effects on the innovation diffusion start to be better explained by introducing a mechanism such that of "word-of-mouth" (WoM) and weak and strong ties (Dichter 1966; Arndt 1967; Granovetter 1973; Brown and Reingen 1987) and social norms and class structure constraint (Reinstaller and Sanditov 2005).

The WoM under strong or weak ties was integrated to the bandwagon behaviour. The bandwagon has been recognised as a distinct behaviour which gives solid

micro-foundation for the interaction. It was first modelled by Leibenstein (1950) based on previous contributions of past economists, psychologist and sociologists, such as Veblen (1899), Rae (1905) and Morgenstern (1948), in the phenomena of fashion and conspicuous consumption. It is a special case of non-additivity in utility functions such as the Snob and Veblen effects. The use of bandwagon preferences in innovation literature is usually associated with threshold models of innovation diffusion such as Granovetter (1978), Rosenkopf and Abrahamson (1999), Acemoglu, Ozdaglar, and Yildiz (2011) and many others[3]. In these models agents only adopts a new product influenced by society when a certain amount of others have done it already. However, as pointed out by Valente (1996) "individuals vary in their willingness to take risks in adopting a new idea, product or behaviour before everyone else", and so diffusion of a product might be unpredictable if information about the distribution of individuals profiles towards innovation is scarce. In this case, the figure of representative consumer fails to describe or predict the aggregate dynamic transformation of a society.

Since the time that Erdös (1947), Erdös and Rényi (1959) and Erdös and Rényi (1961) have developed a theory and a mathematical way to build and typify the properties of graphs or networks the study of interactions between agents has gained momentum and has been applied to a vast list of economic problems such as international trade, occupational mobility, innovation diffusion, urbanization and welfare, financial markets and many more[4].

When modelling innovation diffusion through bandwagon behaviour economists and sociologists usually make use random graphs and small world networks (SWN)[5] as a framework for the analysis, such that developed by Watts and Strogatz (1998) and Watts (1999). A remarkable feature of the Watts and Strogatz's network is that it can rewire the links without lost its topological properties in such a way that the network is neither completely random nor deterministic. This type of graph is well suited to model many social and economic phenomena and it is very popular for being an elegant framework to capture individual bounded information[6] as it supports both linear and stochastic threshold models of the bandwagon.

Nevertheless if one considers that both consumers and firms have bounded rationality, awareness and social mobility[7], simulation of traditional threshold models of bandwagon starts to become a challenge since the probability to adopt new products will depend on the network topology and threshold parameters. Some progress has been achieving by Blume (1993) and Ellison (1993) to endogenize the threshold who developed a stochastic feedback process in which the probability that a given individual adopts the behaviour of his neighbours at a given time is an increasing function of the number of his neighbours who have adopted it and thus shifting the threshold values from deterministic to stochastic and allowing a more suited way to treat the contagion

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[3]For a comprehensive review of innovation diffusion literature, especially that using agent-based models see Kiesling et al. (2012).

[4]For a longer list see Wasserman and Faust (1994, p. 5 e 6) and Benhabib, Bisin, and Jackson (2010b,a)

[5]The so-called *Small Word Network* was coined by Milgram (1967) and Travers and Milgram (1969). According to Milgram, the world is not small because of its physical size, but the people or agents are connected to each other so that you can reach someone else in the other side of the planet using few links in a network of relationships. The impressive figure found by Milgram was 5.2 connections in average in an experiment between dwellers of Nebraska and Kansas in EUA.

[6]A perfect information environment would be nothing more than the smallest world possible, with everyone directly knowing (being connected to) everyone else. The network degree, in this case, is equal to 1.

[7]We mean by social mobility as a dynamic environment where agents, consumers and firms, freely rewire its links or connections in a network of social relationship according to their own micro-motives. The network, in this case, is a consequence of this affluent world filled with a myriad of individual decisions.

process of agents with bounded awareness[8]. Notwithstanding, the network's topology remains to be micro-founded.

The consumer's profile toward a new technology is an important feature of the diffusion process. Goldenberg, Libai, and Muller (2010) made clear the chilling effect on growth when the early adopters are late in a word-of-mouth world. The bandwagon contamination can delay starting without them. The Goldenberg's model was transposed to a small world network and extended by Mukherjee (2014) who demonstrated that the size and the average degree of the network can offset the chilling effect on diffusion of new products. A remarkable feature of this models, and those surveyed by Kiesling et al. (2012), is that the consumer's interaction happens inside a Watts and Strogatz (1998) pre-defined small world network (SWN). All these models assume exogenous networks.

An exogenous SWN, however, while suitable to analyse the process of the diffusion, incurs on limitations when the analysis needs to be further extended e.g. if one wants to develop a more complex model to capture the behaviour of the agents and decision-making processes which generate the network. This holds true because of the fact that agents do not have mobility and follow previously connected links inside a SWN such that developed by Watts and Strogatz (1998) and Watts (1999). The possibilities for modelling interactions with other types of agents or with the environment are then severely limited. In these models, the topology of the network is exogenously imposed to the agents, who follow and obey its properties behaving automatically like a social ant or a termite. Where that topology comes from remains an open issue.

To surpass these limitations we combine the stochastic threshold idea with consumer profiles to develop a complex agent-based model in a graph framework in which the edges are dynamic and micro-founded and the acquaintances (Pool and Kochen 1978) or links depend on the radius of consumer's awareness defined as a radius of perception around a node (consumer). The network topology comes from the behaviour of consumers motivated by their awareness. The links are constantly recreated while agents move around. Consumers probability to adopt a new product depends both on their profile towards new technology, on the number of neighbours who already adopted and on the price of new products set by firms[9].

The diffusion literature has classified the consumers based on their innovativeness profile i.e. the point in time they decide to adopt the new product or technology. The original study of Ryan and Gross (1943) who first classified the behaviour of consumers for this purpose has inspired many recent threshold models like Rogers (1995) and Valente (1996) that define four categories: early adopters, early majority, late majority and laggards.

A recent empirical study performed by Moore (2005) finds percentages for each one of the consumer profiles in the market. Have you ever seen those hundreds of kids camping outside malls to be the firsts to put their hands on new iPhones, or to be the first to read a new episode of Harry Porter? According to the author they are called *techies* and correspond to about 2.5% of the consumers. The following profiles are similar to Rogers and are presented in decreasing order of appreciation towards new technology or products: *visionaries* (12.5%) may not buy the product at the time of the release but they have a high probability to buy it soon. *Pragmatists*

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[8]Additionally, see Chugh and Bazerman (2007)

[9]There are other possibilities such as the use of evolutionary NK models. See Valente (2008) for more information on NK-based models, where agents walk around on a landscape scanning for what seems to be evolutionary better for them within a radius of vision.

(35%) and *conservatives* (35%) correspond to the bulk of the population and are similar to early majority and late majority profiles. Finally, *skeptical* (15%) rarely care for new products even when the majority of the population owns it. The Moore's study was based on the EU market, but of course, another society can show a different behaviour. Our model allows analysing how innovation can diffuse in different populations considering the Moore classification (see section 5.4).

Nevertheless, the progress obtained by these network-based models, many micro behaviours and market dynamic remains to be better explained, what can be done by relaxing the network topology and making it endogenous relative to the consumer's behaviour. We can transform an SWN in a *small live world network* (SLWN).

## 3.   Small Live World Network, Bandwagon and Satiation

Although the model to be built does not account for a formal utility function nor does it requires utility optimization, each consumer has an individual preference towards the adoption of more technologically advanced products, given by Moore (2005)'s consumer classification. To make a decision the consumer considers his or her preferences as well as the network externalities component given by the potential interactions with another agent within the same awareness radius. Heterogeneous consumers make their adoption decision based on their personal preference towards new technologies and the network effect of being convinced to delay or to hop on the bandwagon, depending on the profile of other consumers inside the perception field. That environment generates a world of interactions with remarkable significance for micro and macroeconomic thinking, as will be shown in section (5).

Going slight beyond the limited SWN we propose a new approach to agents interaction in which agents' (nodes) behaviour are not static or responsive to previously defined topology. We assume the consumers randomly move across a bi-dimensional wrapped[10] plain with size $\Gamma$ and dynamically interact with each other according to a radius of conscience which we name *awareness radius*$= \Lambda$. The awareness radius determines the acquaintance volume (Pool and Kochen 1978) of the interaction of the agents. By doing so, the agents are always changing their relative positions to other agents and thus have the opportunity to interact with different sets of neighbours at each point in time with a range defined by their awareness. The neighbourhood of a given consumer $i$ at time $t$, in this case, is defined not by previously randomized edges but by the set of other agents currently inside consumers $i$ awareness at time $t$. We consider that this type of interaction forms temporary local weak ties (Granovetter 1973). Rather than physical topology, the space $\Gamma$ can represent all the behaviour space of consumers, especially the capacity to make connections as explained better in the next section.

In contrast with static SWN environment, we named this new dynamic network as *Small Live World Network* - SLWN. Our network differs from previous ones in the sense that the topology in our network comes from the potential interaction formed by neighbouring agents with temporal and local weak ties.

Let's define the awareness of an agent $i$ as a circle of radius $\Lambda$ around the centre of agent $i$ that captures his capability to perceive and draw information from other

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[10]When an agent cross any of the boundaries of the plain he arrives at the other side. Topologically it is the same to say that the agents inhabit and walk on a surface of a sphere in an elliptic geometry due to Lobachevsky and Bolyai, but projected in a Cartesian plane.

agents and from the environment around him[11]. Note that the concept of awareness here, although homogeneous to every consumer, does not relate to a global information spreading mechanism such as in Bemmaor (1994). Imagine an individual in the subway heading to work. He might be paying attention to the people standing or sitting next to him but he might also be distracted drowned in thoughts about job or family or just texting something on his cellphone, for example. These two different scenarios yield two different awareness radius. In the second case, for example, he might miss the fashionable new high-tech smartphone carried by another individual in the same wagon sitting a bit distant from him, but he will probably notice it if the individual is standing or sitting right next to him.

Consider now that he did see the other individual smartphone. It might fit his personal technological tastes perfectly, causing him to rush to a smartphone store to buy it on the very same day, especially whether he is a *techie* or *visionarie*. But he could be weakly impacted if he was *conservative* or *skeptical*. In this case, awareness combined with heterogeneous behaviour played an important role on innovation diffusion even without any social pressures, because the adoption of the new product by our test subject happened exclusively because of his personal tastes. Now imagine that our subject is fond of technology, but not so desperate to acquire new expensive products right when he sees it. As time passes and he catches the same train to go to work or any different train to get wherever, chances are that he will see more and more different people wearing the same phone and might decide that is time to switch his outdated phone by a state of the art one. This is captured by the stochastic characteristic of the adoption process in our model. The more adepts a consumer perceives through time the higher the chance that he will also become an adept.

The little wagon interaction described above, if the readers pardon the pun, yielded a bandwagon behaviour, but one that has little to do with the strong ties of his social network[12].

The framework in figure (1) shows how this small live world network looks like and works. The figure is a snapshot taken at time $t$. This small world has a surface of size $\Gamma$ inhabited by a population of $\Omega = 10$ agents. In the simulation ahead, this population shall be set to 100. We say *live* because firms can enter and bankrupt in the market while consumers can be born, walk around, make decisions by interacting with others inside their awareness radius $\Lambda$ and die. But the main reason we call or make this network *live* is because that the edges connecting a consumer with others and with firms are extremely dynamic. The edges are created and destroyed as the consumers move around. An edge for the consumer $i$ is created when a consumer or a firm goes into the radius of the reference consumer. This is rather different compared with SWN where the edges are exogenously and *a-priori* created. In our case, the edges vary across time and depend on the simultaneous movement of other agents so their amount at a given time varies positively with the awareness $\Lambda$ of consumers.

Awareness can be heterogeneous regarding individuals and time, but in the simulations presented further ahead, we test only different degrees of homogeneous awareness. Since the edges are created and destroyed endogenously, an important effect of interaction between consumers and firms arise in a complex manner: the possibility

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[11]The another advantage of the SLWN model is that, since there are no directed or undirected edges, it allows for easy modelling of different influential weights between agents in the same neighbouring set.

[12]One can argue that the concept of social networks might cover the set of all people a given agent meets, but that is quite unrealistic and broad. Besides this we could move away from the wagon to the Facebook or another social network where you can decide who goes into your friend list, changing your "electronic" awareness radius.

**Figure 1.:** Small Live World Network
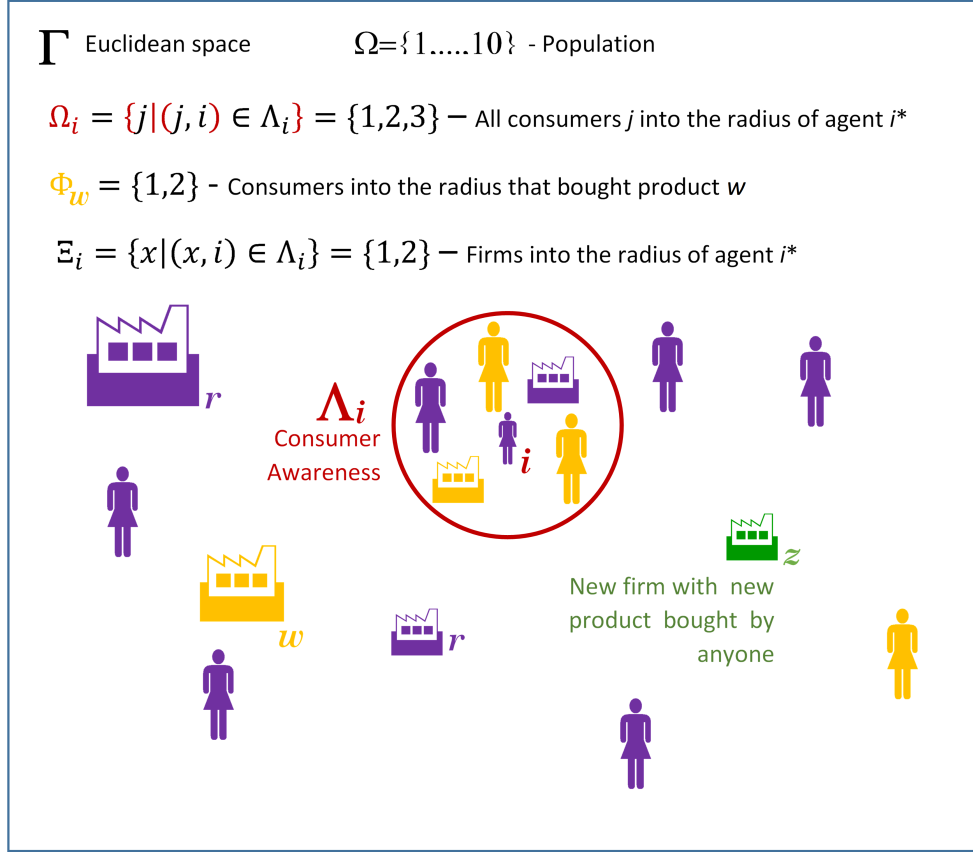


of a cascade effect which can be appropriately called as *bandwagon effect.* Bandwagon pressures may arise from both the total amount and the percentage of people who adopted a certain product in the neighbouring set.

Following figure (1), each of the 10 consumers in the plain has a radius awareness in which a number of other consumers and firms are temporarily located. In the case of consumer $i$, the subset of consumers inside his radius has a cardinal measure of $|\Omega_i| = 3$, while the subset of firms in the same condition has a cardinal measure of $|\Xi_i| = 2$. Each of these agents provides different information about the market. Suppose that two of the consumers, the yellow ones, have already bought a product $w$ of better quality, while the purple ones (including consumer $i$) owns product $r$, of inferior quality. So we have a subset $\Phi_w$ of consumers who have already bought product $w$ and can potentially haul consumer $i$ to this new product convincing him to take part of the crowd who follows this bandwagon. Having been exposed to a new product $w$ for the first time, consumer $i$ faces the decision process of adopting the new product based on his personal preferences towards new technologies, price, fads, fashion and several other possibilities regarding behavioural traits. Note that there exists also a new firm (green) on the market that has recently successfully innovated, creating a new product $z$, that no one has bought it yet.

Consider a set $\Omega$ of consumers with heterogeneous preferences towards the adoption of new technologies, given by $\beta$, and homogeneous awareness radius $\Lambda$ that are randomly spread across a bi dimensional wrapped plain $\Gamma$. Consider also that the set of consumers who already adopted a recently released new technological product

$w$ is given by $\Phi_w$. At any point in time $t$, the neighbouring set of individuals who can potentially influence a given consumer $i$ is given by $\Omega_i(t) = \{j \mid (j,i) \in \Lambda_i\}$, where $j$ comprises all of the individuals in a range of consumer $i$ awareness. The consumer is influenced both from the amount and percentage of neighbours who have already adopted. If we take a cardinal representation of sets and subsets[13], we can represent the decision process as follow:

$$\omega_i \in \Phi_w(t+1) \quad if \quad \theta_i^q \equiv \beta_i \left( \alpha \frac{|\Phi_w(t) \cap \Omega_i(t)|}{|\Omega_i(t)|} + (1-\alpha)\, |\Phi_w(t) \cap \Omega_i(t)| \right) > \epsilon$$

where $0 \leq \alpha \leq 1$ is a constant denoting the weights for total and percentage of adopters and $\epsilon$ is a random number drawn from a uniform distribution between 0 and 1. The equation means that the consumer $i$ decide to adopt new product $w$ if the condition in the first part of the equation is higher than $\epsilon$, and therefore, he will belong to the set $\Phi_w$ in the next period and thereafter will transmit his new experience to others as time passes. We will return to this equation further up.

This bandwagon behaviour generates a diffusion process towards a market saturation. Figure (2) below shows the simulation results of the diffusion pattern of the model for different consumer awareness radius values. We can clearly see that innovation diffusion follows the traditional Product Life Cycle S-shape and that as rationality (awareness) increases so does diffusion.

**Figure 2.:** Dynamic Innovation and Saturation in the PLC theory
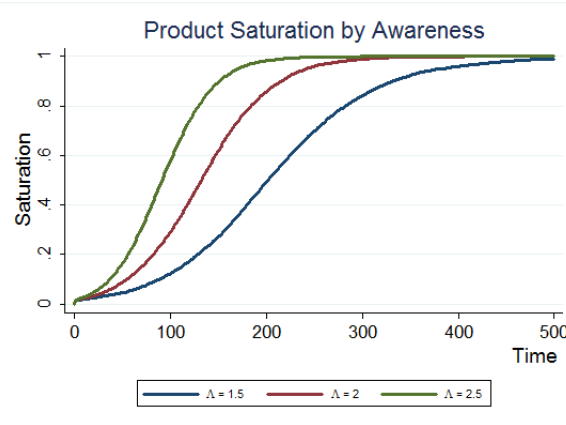


The life cycle is not by any margin an exclusive economic phenomenon. From natural to social sciences, from bacteria to sales, the majority of growth processes follow the same pattern of birth, acceleration, dis-acceleration and decline, generating an S-shaped curve. It is no different in economic regimes. Using the words of Aoki and Yoshikawa (2002, p. 127), "plot a time series of production of any representative product such as steel and automobiles, or production in any industry, against year, and, with few exceptions, one obtains an S-shaped curve".

It is not difficult to understand the mechanism behind a S-shaped curve. In fact, the key words here is also one that motivates economic science: scarcity and saturation. In the beginning, there are almost no constraints for growth, be it space, food, knowledge,

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[13]The bar surrounding the numerator and denominator of the first term on the left denotes cardinality and represents the number of items inside each set respectively.

demand: they are usually abundant at first so the process can grow exponentially with no boundaries. But when it reaches a certain stage scarcity begins to take its toll and the agents involved start to feel the lack of resources, preventing them from freely "reproduce". The growth then begins to slow down until it reaches a maximum possible equilibrium or to decline if it cannot maintain itself at a given level for internal or external reasons. Scarcity restricts the growth by the lack of resource by the supply side. Saturation, on the other hand, accrue from the behaviour of consumers subject to the forces of satiation. The amount of product a consumer or society is able to consume is limited, even when it could be abundant without any resources restrictions. For example, a given product can never surpass a certain level of sales even if its price tends to 0 because there is a maximum amount of people in the market. Sales and consumption will never grow to infinity.

A new product diffusion in the Product Life Cycle takes place mainly through a word of mouth process (Czepiel 1974) that closely resembles bandwagon behaviour. Consumers interact with each other and spread the rumour of the new product accelerating the diffusion process. The only difference between the concept of word of mouth and bandwagon is that in the latter there is no need to interact with others to be influenced by them. The mere fact the individuals are able to see what kind of behaviour, costumes and products society is taking or consuming is a sufficient condition for bandwagon contagion. As the saturation of a given product approaches its limit, the firm becomes more and more inclined to try and perform a product innovation with the aim of igniting a re-buying process leading to a new S-shaped cycle of saturation and maturity. This process is described by Higuchi and Trout (2008) as *De-maturity*, in which the firm prevents the arrival of the maturity phase of the PLC by creating a new product version.

## 4. A complex small live world model

In this section, we present a formal agent-based model where consumers have bounded awareness and sequentially make not maximizing decisions about buying only one product from the spectrum of available firms. Consumers are heterogeneous with respect to what we call *profile* towards new products and follow the distribution proposed by Moore (2005) as a baseline. Since our consumers do not have information about global demand or preferences, the probability to change consumption choices plays a stochastic role in traditional deterministic thresholds. Consumers can only get information about quality and prices by interacting with each other when they are inside each other's awareness radius[14]. Consumers then evaluate how many others are using a given product or technology and decide to change their product of choice or not with probability $\theta_i^{p,q}$, as he is influenced by price or quality. Firms, on the other hand, cannot see individual preferences, but analyse market saturation and concentration to decide on the amount of R&D investment on each period $t$ and between process and product innovation. There are strong interactions between consumers and consumers, and consumers and firms which produce complex dynamics of innovations, cycles and economic development.

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[14]Similar to Chugh and Bazerman (2007) and Ellison (1993).

## 4.1. *Consumer Interaction and Bandwagon Behaviour*

Consider a constant length set of consumers given by $\Omega = \{1, ..., N\}$ with heterogeneous profiles (propensity to assimilate new technologies or products) that follows a quasi-normal distribution in respect to their probability of incidence in the population described in table 1 below. Each consumer $\omega_i$ carries an intrinsic propensity to adopt new products according to their profile, which we call stochastic bandwagon threshold ($\beta$). Consumers are randomly spread and randomly move across a bi-dimensional wrapped plain $\Gamma$ characterizing our economic space.

**Table 1.:** Bandwagon Population

| Type | % of population | $\beta$ |
|---|---:|---|
| Techies | 2.5 | 1 |
| Visionaries | 12.5 | 0.8 |
| Pragmatists | 35 | 0.6 |
| Conservatives | 35 | 0.4 |
| Skeptics | 15 | 0.2 |

Consumers have the same bounded awareness represented in the model by a circle of radius $\Lambda = \{1, ...\Gamma/2\} \subseteq \Gamma$ around them[15].

Consumers can also die with the probability $k_4 = [0, 1]$, but they immediately give birth to another agent with the same profile, the only difference being that new consumers enter into the space $\Gamma$ with no supplier[16]. This mechanism is introduced in order to capture the effects of new agents constantly entering the market. since they have no information whatsoever about products prices and quality they are important to contribute to smaller firms demand and thus prevent monopoly situations. The set of consumers at time $t$, $\Omega = \{1, ...., N\}$ as well as its distribution is, nevertheless, always kept constant[17]

At any point in time $t$, consumers may own only one product $z$ of quality $q$, supplied by firm $x \in \{0, ..., X\}$ for which they pay the price $\rho$[18]. However, they can be influenced to change their adopted product if they are inside each other's awareness radius i.e., if they are neighbours with the relative position between $(\omega_i, \omega_j) = (0, 0...\Lambda)$ in the plain. In this coordinate 0 means the reference point where the consumer stands, from which he or she can see around by a distance of radius $\Lambda$. The neighbouring set of individuals who can potentially influence consumer $i \in \Omega$ in time $t$ can then be described as $\Omega_i(t) = \{j \mid (j, i) \in \Lambda_i\}$, where $j$ comprises all of the individuals in a range of consumer $i$ awareness ($\Lambda_i$). The consumers $j$ inside the view radius of consumer $i$ carry some information which can convince or not he or she to buy the new product. For example, it can happen that a consumer $j$ be a *skeptic* who has not bought a new technology yet and, therefore, do not carry information about quality $q$ and price $\rho$ of a new product.

We assume that at the initial state of the model $t(0)$ there is only one incumbent firm that invents a brand new type of high-tech product and immediately engages in marketing activities capturing all consumers with a *techies* profile. These "crazy

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[15]Note that this is not the same radius suggested by Young (2002) in his model where agents are distributed along the edges of a radius.

[16]And consequently without a product, quality, the price paid and so on.

[17]This is a necessary condition to analyse how changes in the distribution (more precisely the number of *techies*) affects the evolution and final stage of the model during simulation.

[18]We choose to make two distinctions about notation: between the price paid by a consumer $\rho$ and the price charged by a firm $p$; and between the consumer product provider $\chi$ and the firm per se $x$.

for technology" consumers will also instantly buy any new better quality versions of the product provided they are clients of the innovating firm at the time of product innovation[19]. At time $t$ the subset of consumers who adopted a given product $z$ in the market is given by $\Phi_z(t) \subseteq \Omega$.

A consumer $i \notin \Phi_z(t)$ may be influenced[20] by his neighbours to change his product in two distinct situations: when faced with one or more neighbours $j$ who have adopted: a) a better quality product[21]; or b) a product of the same quality from a firm with a lower price.[22] The cyclical sequence of steps of the simulation model is described in figure 3 below:

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[19]Techies will always act as seeders of the new versions of the product.

[20]Consumers may only be influenced once per time $t$.

[21]If there is more than one different product quality perceivable inside $\Lambda$, Consumer $i$ will only consider a change for the best one.

[22]Pay close attention to the fact that consumers will always prefer a quality change over a price change in the form: $q \succ \rho \Leftrightarrow (q \succ \rho \bigwedge q \nprec \rho)$. i.e., there is no utility function in the model in the form of a trade-off between quality and price because consumers are assumed to have enough income to buy any product they want. No income restriction is a strong assumption and needs to be addressed, but for the moment we decide to make the model as simple as possible to focus on the complex dynamics of awareness, populations and firms strategy. The relevant issues on the personal income distribution is a non-trivial issue in passing to be attacked on another occasion.

**Figure 3.:** Sequential Steps of the Simulation Model
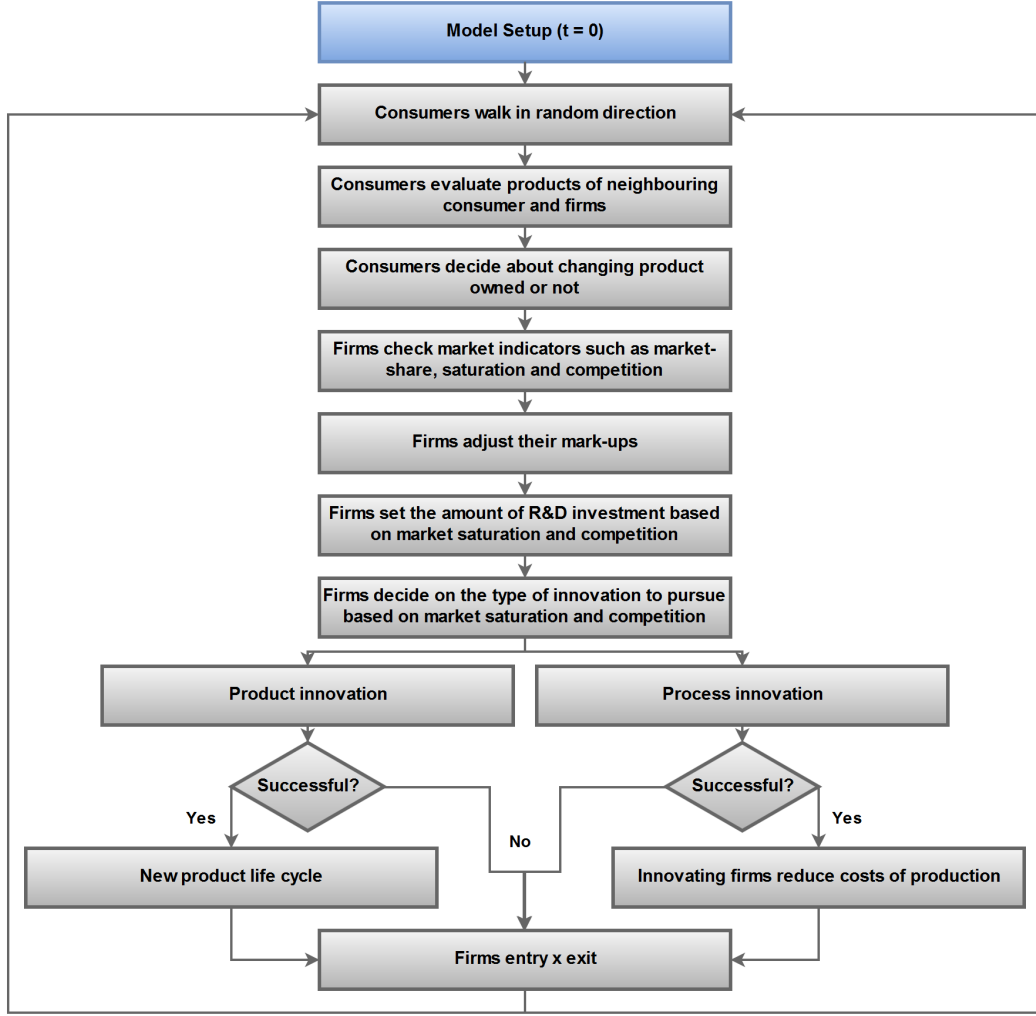


### 4.1.1. Adoption by quality

Start by addressing the first case. A consumer $i \notin \Phi_z(t)$, walking randomly through our plain, meets in his awareness radius $\Lambda$ at least one neighbour who owns a better quality product $z_j > z_i$. The probability that consumer $i$ will be influenced to buy the better-quality product he saw will be affected by three factors: the amount and percentage of the neighbours owning that product at time $t$ and consumers $i$ bandwagon profile. The decision process is detailed below. Note that the bars delimiting the terms in the numerator and denominator of the first term of the equation denote cardinality i.e. the number of agents on each of the sets. The decision process is given by:

$$\omega_i \in \Phi_z(t+1) \quad if \quad \theta_i^q \equiv \beta_i \left( k_1 \frac{|\Phi_z(t) \cap \Omega_i(t)|}{|\Omega_i(t)|} + (1 - k_1) \frac{1}{1 + e^{-|\Phi_z(t) \cap \Omega_i(t)|}} \right) > \epsilon \quad (1)$$

where $\epsilon$ is a random number following a continuous uniform distribution over the range $\epsilon = U[0,1]$ and $\{0 \leq k_1 \leq 1\}$ is a constant measuring the impact of each type

of influence[23].

Now consumer $\omega_i$ is left with the decision of where to buy the product, i.e. from which one of the available firms. If there is only one candidate $j \in \Lambda_i(t)$ for potential influencer in consumer $i$ awareness radius $\Lambda_i$, the decision is straightforward and he will buy from his neighbour's supplier so that $\chi_i(t+1) = \chi_j(t)$. However, in cases where the subset $\Phi_z(t) \cap \Omega_i(t)$ is non-unitary, it may be possible that there will be two or more neighbours who are clients of different firms. How will consumer $i$ choose between the available firms? He will choose to buy the selected product from the firm that incurs him the lowest effort $e_i$. The effort of a given consumer $\omega_i$ to buy a product $z$ from a provider firm $\chi$ is given by:

$$e_i = \rho_{i,\chi} + k_2 d_{i,\chi} \tag{2}$$

where $\rho_i$ is the price paid by agent $i$ for product $z_i$ of firm $\chi_i$ at time $t$, $d_i$ is the distance between the consumer $i$ and his supplier $\chi_i$ and $0 \le k_2 \le 1$ is a constant measuring the impact of distance on a consumers effort.

But consumer $i$ does not have information about the prices of all firms in $\Gamma$, therefore he will need to choose based on the effort information gathered from his neighbours $j \in \Lambda_i(t)$. The firm $\chi$ choose by consumer $i$ will depend on the interacting process given by:

$$\chi_i = \arg\min_\chi \{e_j(\chi_j) \mid \chi_j \in \{1, ..., X\}; j \in \Lambda_i(t)\} \tag{3}$$

where $e_j(\chi_j)$ is the effort the consumer $i$ must incur to reach a firm $(\chi_j)$ within its view radius $(\Lambda_i)$.

### 4.1.2. Adoption by price

Now consider the second set of interaction between consumers in which $\Phi_z(t) \cap \Omega_i(t) = \emptyset$. In this case, consumers bandwagon behaviour neither the amount and percentage of neighbours who own a product with a lower price matters because they are not analysing a change in product quality/technology. However, we assume that there is an associated cost of changing the firm providing products for consumer $i$ and thus changing is not guaranteed but depends positively on the difference between the price paid by him $\rho_i$ and by consumer $\rho_j$[24]. Consumer $i$ will ultimately change his supplier $\chi_i$ if his new effort $e_i(\rho_j, \chi_j)$ is lower than the current one $e_i(\rho_i, \chi_i)$. The interaction results in:

$$\chi_i(t+1) = \chi_j(t) \quad \text{if} \quad (1 - \frac{\rho_j}{\rho_i}) > \epsilon \quad \text{and} \quad e_i(\rho_j, \chi_j) < e_i(\rho_i, \chi_i) \tag{4}$$

Besides being influenced by neighbours, consumers can also be influenced by firms inside $\Lambda_i$. The set of firms that can influence consumer $i$ at time $t$ is given by $\Xi_i(t) = \{x \mid (x, i) \in \Lambda_i\}$[25]. Similarly to the pure consumer interaction there are two ways in

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[23]From here on we will drop the time notation when it is not necessary.

[24]As in the previous case consumers will only consider being influenced by his neighbour with the lowest price.

[25]Note, though, that individuals cannot interact with buildings so we assume that the interaction occurs between a consumer and a salesman trying to convince him to buy the latest best-quality product of the firm.

which a firm may influence a given consumer $i$ to change his product: a) there is at least one firm $x$ in $\Lambda$ selling a product with a better quality than $z_i$; and b) there is at least one firm offering a product with the same quality as $z_i$ but for a lower price.

In the first scenario, consumer $i$ finds a firm $x$ with a better quality product $z_x > z_i$ than he has and is influenced to buy it. The decision whether to adopt or not adopt this product and supplier will depend solely on consumer $i$ bandwagon profile according to:

$$z_i(t+1) = z_x , \quad \chi_i(t+1) = x \quad \text{if} \quad \beta_i k_3 > \epsilon \tag{5}$$

where $0 \leq k_3 \leq 1$ is a constant.

If the set $\Xi_i(t) = \{x \mid (x, i) \in \Lambda_i\}$ is non-unitary, there might be two or more firms charging different prices $p_x$ for products with the same quality. In this case, consumers will always prefer the firm offering product at the lowest price:

$$\chi_i = \arg \min_x \{p(x) \mid x \in \{1, ..., X\}\} \tag{6}$$

The second setting presents the case in which a consumer owning a product with quality $q_i$ finds a firm $x$ selling the same product for less money i.e., $p_x < \rho_i$. Analogously to the pure consumer interaction consumer $i$ will decide between changing his supplier or not based on the difference between the price he is paying for the product and how much the firm is selling it for:

$$\chi_i(t+1) = x \quad \text{if} \quad (1 - \frac{p_x}{\rho_i}) > \epsilon \tag{7}$$

Although the model does not account for a formal utility function nor does it requires utility optimization, each consumer an individual preference towards the adoption of more technological advanced products, given by Rogers (1962) and Moore (2005) consumer classification, as well as a homogeneous network externalities component given by the potential interactions with another agents inside the same awareness radius. Consumers make their adoption decision based on their personal preference towards new technologies and the network effect of being convinced to hop on the bandwagon.

### 4.2. Firm Behaviour

As mentioned in the previous section, at time $t(0)$ only one incumbent firm[26] creates a new economic sector or market[27] by inventing a product and immediately captures the demand of *techies*. From this point on new firms can enter and exit the market if they find it profitable, considering the option of investing in bonds paying the constant interest rate $r$. Thus at a given time $t \neq 0$ there will be a set $\Xi = \{1, ..., X\}$ of firms in the market. Firms produce goods on-demand and thus doesn't accumulate stocks.

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[26] Adding more incumbent firms do not change the outcome of the model except for the initial HHI concentration index.

[27] The new sector can be view both as industry or service.

There is also no kind of economies of scale or scope i.e., cost reduction arises only from successful process innovations.

There are two pivotal decisions a firm has to make in the model: how much to spend in research and development activities and what type of innovation to pursue given the market situation at any point in time. Based on the literature, we assume that market saturation and competition play a key role in the firm innovative decisions.

### 4.2.1. R&D expenditures decision

Despite Schumpeter's claim that monopoly power provides incentives for innovation by raising the firm's capabilities, recent literature on the subject[28] shows that it is usually competition that fuels innovation. The argument is that competition raises the intrinsic costs of falling behind in competitiveness by failing to successfully engage in R&D activities. Aghion et al. (2002) find that these two effects combined generate an inverted-U relationship between the variables. Up to a certain level, competition creates incentives for innovation because firms try to become more competitive reducing costs or differentiating products, but if competition gets too fierce, profits tend to fall along with resources available for R&D. In our model, we assume that investment in R&D is a positive function of competition but since the amount invested is given by a percentage of profits, competition might hinder innovation if it heavily impacts profit.

Pasinetti (1981) has long ago stated that saturation is a natural bottleneck for economic growth and development. Even in the case of a high and sustainable productivity growth, the population would have to grow accordingly for saturation not to happen, which does not seem to be the case for most developed and developing countries. The way firms deal with saturation is through a life cycle de-maturity process of product innovation, especially in durable goods industry where re-buying takes time to occur. McMeekin, Green, and Walsh (2002) provides a great overview of the literature relating aspects of demand with innovation. We use Saviotti and Pyka (2012) equation that relates search activities with market saturation[29] and a similar function to competition impact on innovation to define the percentage of profits a firm will secure for R&D at any point in time:

$$\theta_x = k_4(1 - e^{(-k_5 S_x)}) + k_6(1 - e^{(-k_7 \text{HHI})}) \tag{8}$$

where $\theta_x$ is the chosen percentage of profits secured for R&D, $0 < k_{4,\ldots,7} < 1$ are constants, $HHI$ is the Herfindahl Hirschman Index and $S_x$ is the market saturation for the best-product of firm $x$. $HHI$ and $S_x$ are given by:

$$S_x = \frac{\sum_{i=1}^{N}[C_i \mid z_i = z_x]}{\sum_{i=1}^{N}[C_i \mid z_i \neq 0]} \tag{9}$$

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[28]For more details about the relationship between competition and innovation see Aghion et al. (2002), Aghion et al. (2006)

[29]It is important to note that firms will always consider the saturation of their best-quality product for innovation decisions and not the global saturation level

$$\text{HHI} = \sum_{x=1}^{X} ms_x^2 \tag{10}$$

where $ms_x$ is the market-share of firm $x$. Thus the investment in R&D of firm $x$ at time $t$ is given by:

$$I_x^{R\&D} = \theta_x \pi_x \tag{11}$$

with

$$\pi_x = \sum_{z=1}^{Z} (D_x(p_x - c_x) \tag{12}$$

where $\pi_x$ is the profit, $D_x$, $p_x$ and $c_x$ the demand, price and cost for each product of firm $x$.

### 4.2.2. Innovation type decision

After deciding on the number of R&D resources, firms have to decide in what type of innovation they are going to invest these resources. It is reasonable to assume[30] that market saturation has a bigger impact on product innovation than process innovation because a firm can only overcome saturation with process innovation while there is still a potential demand to be conquered. However, it is a little more complicated to infer the contrary for market competition, since both product and process innovation can help a firm to gain a competitive advantage[31]. Nevertheless, we assume that firms will prefer to invest in product innovation only when market saturation is higher than the market competition and saturation is above a certain threshold $S_{min}$. This is due to the fact that product innovation is considered to be more difficult to successfully implement than process innovation since it requires consumers approval.

$$\text{Type} = \begin{cases} \text{Product} \Rightarrow I_x^{R\&D}(t) = I_x^{PD}(t) & \text{if} \quad S_x(t) > H(t) \quad \& \quad S_x(t) > S_{min} \\ \text{Process} \Rightarrow I_x^{R\&D}(t) = I_x^{PC}(t) & \text{otherwise} \end{cases} \tag{13}$$

where PD means product innovation and PC process innovation. The probability of arrival of an innovation type is a linear function of the total amount of investment in one of them. It follows that:

$$\phi_x^{PD,PC} = k_8^{PD,PC} I_x^{PD,PC} \tag{14}$$

where $0 < k_8^{PD,PC} < 1$ are constants.

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[30] This is a subject yet to be better explored by the literature since results are still inconclusive. Some examples of papers in this area are Bonanno and Harworth (1998) and Rosenkrantz (2005).
[31] See Porter (1999).

### 4.2.3. Effects of successful innovations

We now turn our attention to how a successful innovation changes the innovating firm status. Consider first the case of product innovation. When a firm manages to succeed in product innovation in time $t$, it uses all it's R&D resources of that type[32] to start producing, at $t + 1$, a new product of better quality than its previous best ($Q_{z,x}(t + 1) > Q_{z,x}(t)$), at a higher cost ($c_{Z,x}(Q, t + 1) > c_{Z,x}(Q, t)$) that will be added to the set $\Psi_x(q, t) = \{1, ..., Z_Q\}$ of all products, ordered by their quality, being demanded to firm $x$. The new product is immediately adopted by the firm $x$ set of "techies" clients[33]. We can summarize the impact of a product innovation as:

$$\text{if} \quad \phi_x^{PD} > \epsilon \Rightarrow \begin{cases} Z_{Q,x}(t + 1) \in \psi_x(t + 1) \\ Q_{Z,x}(t + 1) = Q_{z,x}(t) + (\Delta_q) \\ c_Z(Q, t + 1) = (\Delta_c^{PD})c_Z(Q, t) \end{cases} \tag{15}$$

where it is assumed that $\Delta q$[34] and $\Delta_c^{PD}$[35] are constant in time parameters measuring the impact of product innovation on $Q_Z$ and $c_Z$ respectively. The product innovation mechanism above resembles the quality ladder models due to Grossman and Helpman (1991a,b), Aghion and Howitt (1992) and Barro and Sala-i Martin (2004, chapter 7) but differ slightly from the Grossman since the new product do not correspond to a new firm[36]. The entry mechanism will be described further.

Process innovation, on the other hand, has the sole effect of reducing the cost of production of all of a firms products:

$$\phi_x^{PC} > \epsilon \Rightarrow c_{z=\{1,...,Z\}}(t + 1) = \Delta_c^{PC}[c_{z=\{1,...,Z\}}(t)] \tag{16}$$

### 4.2.4. Price setting behaviour

Firms are engaged in Bertrand competition defining their prices primarily by a markup rule, but also considering the mean price of firms in the market when $|\Xi| > 2$. We assume that prices are somewhat sticky, meaning that firms will wait a certain threshold number of periods ($\tau$) before reacting to market-share ($MS_x$) alterations in order to assay if the tendency is persistent or seasonal.[37] The price $p_{z,x}$ charged for product $z$ of quality $q$ by a firm $x$ at time $t$ is given by:

$$p_{z,x}(t) = \begin{cases} \mu_x(t)c_{z,x}(t) & \text{if} \quad |\Xi| \leq 2 \\ k_9\mu c_{z(q),x}(t) + (1 - k_9)\frac{\sum_{x=1}^{X} p_{z,x}}{|\Xi|} & \text{if} \quad |\Xi| > 2 \end{cases} \tag{17}$$

where $\mu_x(t)$ is the markup of firm $x$ at time $t$ and $0 < k_9 \leq 1$ is the impact of markup

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[32]In this case $I_x^{PD}$.

[33]If a firm does not possess any costumer with techies profile the only way it can diffuse the new innovation is through consumer/firm interaction.

[34]We assume $\Delta_q = 10$ although it may take any positive value without changing the outcome of the model.

[35]We follow Adner and Levinthal (2001) and assume $\Delta_c = 0.2$.

[36]The figure (4c) can be compared with Barro and Sala-i Martin (2004, figure 7.2, p. 320).

[37]Due to managerial costs such as menu, customer negotiation and information gathering costs. See Zbaracki et al. (2003) and Dias et al. (2011) for a detailed discussion.

on a firms price decision[38]

Firms alter their markup according to:

$$\mu_x(t) = \begin{cases} \mu_x(t-1) + k_{10}\mu_x(t-1) & \text{if} \quad MS_x(t-\tau) < MS_x(t-\tau+1).... < MS_x(t) \\ \mu_x(t-1) - k_{10}\mu_x(t-1) & \text{if} \quad MS_x(t-\tau) > MS_x(t-\tau+1).... > MS_x(t) \end{cases} \tag{18}$$

where $0 < k_{10} < 1$ is the markup changing reaction.

### 4.2.5. Entry/exit mechanism

At any point in time, a prospective firm faces the decision of entering given the market situation. In order to make the decision, it is assumed that possible entrants observe the mean profit rate of currently operating firms, the saturation of the best-quality product and the concentration of the market. The higher the market profits and concentration and the lower the saturation, higher the probability of enter[39]. Entrants will only be willing to produce the best-quality product in the market at the time of possible entry. Additionally, the saturation of the chosen product cannot be higher than $50\%$[40]. Best-quality product saturation is given by:

$$S_Z(t) = \frac{\sum_{i=1}^{N}[\omega_i \mid z_i(t) = Z(t)]}{N} \tag{19}$$

where $z_i$ is the product being consumed by consumer $i$.

The mean profit rate of firms is:

$$\overline{\varpi} = \frac{\sum_{x=1}^{X} \frac{\sum_{z=1}^{Z} \varpi_{z,x}}{Z}}{X} \tag{20}$$

where $\varpi_{z,x} = \pi_{z,x}/I_x$ is the profit rate of product $z$ of firm $x$.

And thus the decision of a given firm to enter the market is given by:

$$\text{Entry} = \begin{cases} \text{Yes} & \text{if} \quad S_Z < 0.5; \quad \overline{\varpi} > r; \quad \overline{\varpi}S_Z H > \epsilon \\ \text{No} & \text{Otherwise} \end{cases} \tag{21}$$

Firms will exit the market if their mean profit-rate over a range $T$ of periods is smaller than the interest rate $r$, which means firms fight to remain in the market even when profit-rates are smaller than the other possible investment option[41]. At any point in time a firm will check the mean of its last $T$ profit-rates and compare it to the interest rate:

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[38]Simulating the model we will assume that $k_9 = 0.6$ based on studies of Keney, Lawless, and Murphy (2010) who finds that the share of firms that set their prices by markup is about 60% while 40% of firms set their prices by looking at the other firm's prices.

[39]See Dixit (1989) and Chang (2009) for studies about firm entry/exit decision.

[40]We assume a firm will never be willing to enter the market producing a good that is close to maximum saturation because the entrant won't have time to accumulate R&D investment for product and process innovation and will most likely fall behind on technology and product quality.

[41]Is is assumed that there is no depreciation in the model which implies that a firm would be able to fully recover its initial investment $I_x$ if they decided to sell their infrastructure.

$$\text{Exit} = \begin{cases} \text{Yes} & \text{if} \quad \frac{\sum_{t=t-T}^{t} \varpi_x(t)}{T} < r \\ \text{No} & \text{Otherwise} \end{cases} \qquad (22)$$

## 5. Results: some micro behaviour and social consequences of SLWN

The results of the model are presented in four different sections each containing different insights and perspectives in order to maximize the coverage and robustness of the model.

First, the dynamics of the model through a single simulation run, named the baseline run, are explored. Although this approach has to be viewed with careful in regard to its capability to bring conclusive valid results due to the lack of different runs, it is of major importance to unveil the so-called gearbox of agent-based simulations, specially regarding the de-maturity process through which new incremental innovations are released in the market and live their product cycles.

Second, we explore the network topology and average proprieties of the live network which emerge from interacting with consumers given by increasing awareness.

In the third step, robustness is added to variable behaviour analysis by performing 100 different simulations in total, each with a different random seed, comprising 5 different values for awareness and 5 different amounts of techies consumers in the population. Resulting variable values are therefore a mean of these 100 simulations evaluated with respect to time.

Finally, after confirming the robustness, a given random seed is fixed and the values of awareness and the distribution of population are allowed to considerably vary in a given range, resulting in smoothed simulations used to analyse the pattern of product and process innovation as well as the evolution of quality and prices. In this case, the variables are evaluated with respect to the actual awareness and population distribution values.

Table (A2) in the appendix shows information about the values taken by constants in our model, which remains unchanged between scenarios and simulations.

### 5.1. *Baseline run dynamics*

The baseline simulation consists of a unique run of a population according to table (1), in which the distribution of techies, visionaries, pragmatists, conservatives and skeptics are 3%, 12%, 35%, 35% and 15% respectively, and, also, awareness is fixed as 2. Important macro patterns are only observable while considering a single simulation run, which means the resultant behaviour of variables is not a mean from several simulation runs. This is important in order to shed some light on the gearbox of agent-based interactions.

The figure (4) below shows how the de-maturity process takes place in the simulated economy and how some key variables such as quality and costs react to it. At time $t = 0$, a radical innovation is introduced, which means there are no consumers previously buying the product. Techies consumers immediately buy the new technological product and act as seeders for the product life cycle of all versions of the good to be introduced through incremental innovation. Note that saturation of a

product of given quality, represented by the colourful lines[42] in the de-maturity graphic (4a), follows an S-shaped curve as expected from the figure (2). The speed in which how the market saturates can be better represented by the innovation frequency graph (4b). Each time a new product is released by an innovating firm, demand saturation enters a new cycle as consumers are instigated by bandwagon behaviour to update the quality of their product as firms release new goods to respond to market competition (HHI) and saturation. This creates degrees of product quality (graph 4c) and costs/prices (graph 4h) since each product innovation raises the mean cost of production by a small margin.

In the case of the baseline simulation, a faster growth of process innovations (red line of graph 4h) in respect to product innovations is perceivable (black line of graph 4h) which, as will be seen further ahead, is in accord to the other results of the model. It is also possible to see the temporal evolution of other key variables of the model such as the mean costs (black line of 4h), prices (red line of 4h), profits (4g) and markups (4i) amongst firms; and the product innovation profitability, measured by the differences in profit between firms at the technological frontier and the imitators (4f).

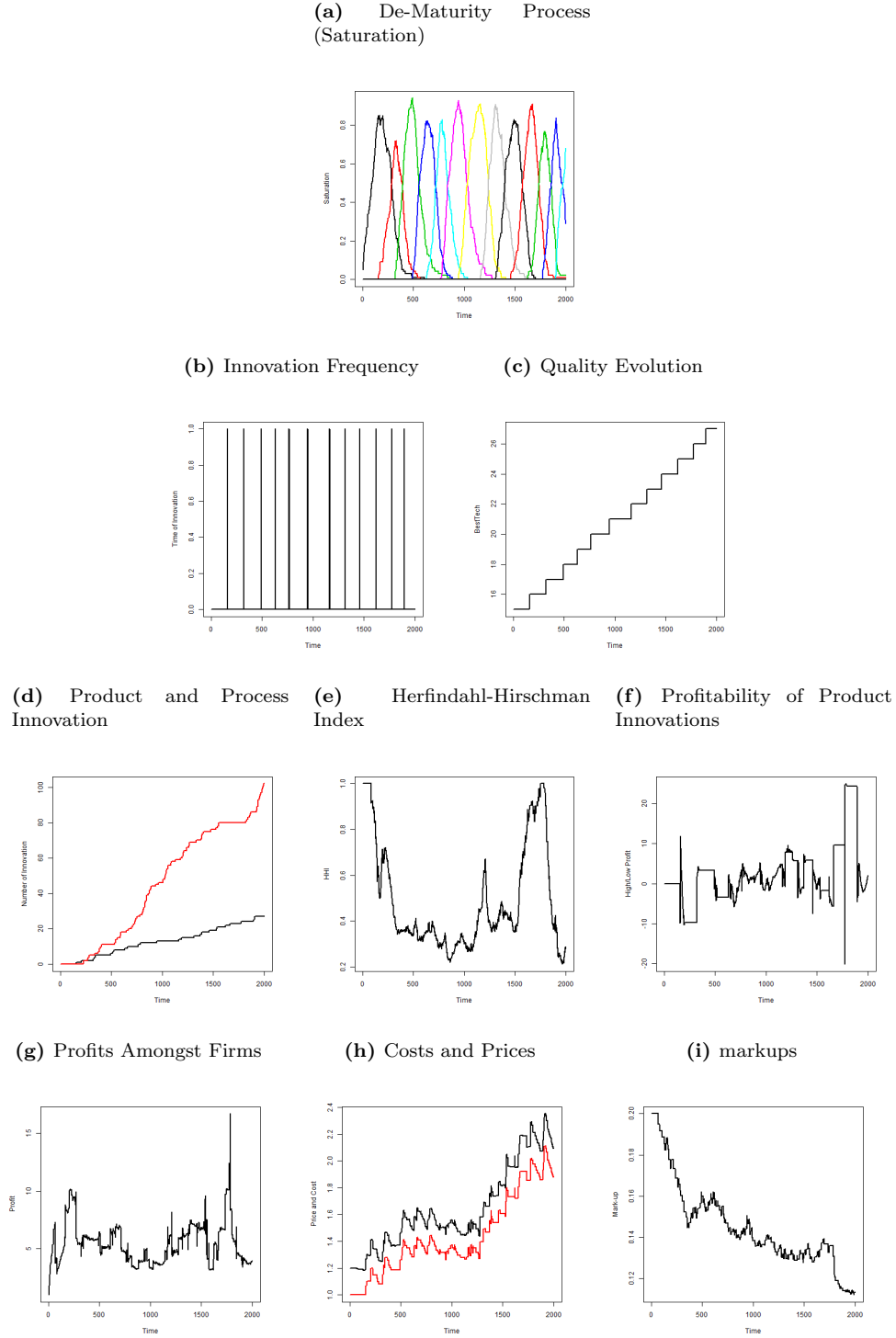## 5.2.  *Average live network topology emerging from increasing awareness*

In this section we provide a more detailed description of the effects of the awareness on the network topology, shedding lights on the consumer behaviour and innovation issues. The main topic here is not about graph theory and small word network only, we just apply this methodology to the case in question and we hope to discuss this methodology in an appropriate occasion with sufficient space for that. A full description of the microeconomic foundations of this SWLN and the network properties and dynamic emerging from that evolutionary microeconomics is a step forward. An important one.

As stated in section (3) the bandwagon effect flows through the small live world network which topology, in turn, strongly depends on the awareness radius $\Lambda$. It is soaring intuitive and effortless to imagine that higher awareness radius makes a consumer more connected with others accelerating the diffusion of a new product in a heterogeneous population. The figure (5) is enlightening to show what happens to the network when awareness radius increase from $\Lambda = 1$ to $9$. Each one of the networks is an undirected graph built from initial conditions of simulations. Despite considering a live network, with a vivid link creation and destruction process, when we allow consumers walk around, the main proprieties of the networks given by a number of links, average degree, density, diameter and many others, remains stable at each $\Lambda$ over time.

Notwithstanding the awareness is crucial to the network topology. It is particularly interesting to observe the relationship between awareness and the diameter of the networks. The diameter is an important property of the SLWN which helps to understand how the awareness draws the network topology and that dings and makes audible the echoes the bandwagon effect. The diameter is the largest distance between two nodes in the network, measured by the number of edges existing between the two nodes. At low awareness, all nodes (consumers) are disconnected like in figure (5a) where the diameter is only 3 and happens only once. Nothing echoes here and this small network is a silent world. Increasing awareness generates denser networks with

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[42]Only the firsts product qualities are visible.

**Figure 4.:** Temporal Evolution of Variables in the Baseline Simulation

**(a)** De-Maturity Process (Saturation)



**(b)** Innovation Frequency

**(c)** Quality Evolution



**(d)** Product and Process Innovation

**(e)** Herfindahl-Hirschman Index

**(f)** Profitability of Product Innovations



**(g)** Profits Amongst Firms

**(h)** Costs and Prices

**(i)** markups



more connected nodes but at cost of long distance or higher diameter. That is what happens to the awareness 4 in figure (5d) when diameter reaches 19 edges. In this case, the echoes reverberate slowly at a long distance. After awareness 4 the diameter starts to decrease, making the network more connected and nodes who was in the extreme

22

**Figure 5.:** Average Live Network Topology by Awareness



**(a)** Λ = 1.0

N° edges = 20
Av. Degree = 0.396
Diameter = 3
Density = 0.002

**(b)** Λ = 2.0

N° edges = 68
Av. Degree = 1.347
Diameter = 7
Density = 0.007

**(c)** Λ = 3.0

N° edges = 132
Av. Degree = 2.614
Diameter = 10
Density = 0.013

**(d)** Λ = 4.0

N° edges = 216
Av. Degree = 4.277
Diameter = 19
Density = 0.021

**(e)** Λ = 5.0

N° edges = 331
Av. Degree = 6.554
Diameter = 14
Density = 0.033

**(f)** Λ = 6.0

N° edges = 482
Av. Degree = 9.545
Diameter = 7
Density = 0.048

**(g)** Λ = 7.0

N° edges = 660
Av. Degree = 13.069
Diameter = 5
Density = 0.065

**(h)** Λ = 8.0

N° edges = 842
Av. Degree = 16.673
Diameter = 4
Density = 0.083

**(i)** Λ = 9.0

N° edges = 1075
Av. Degree = 21.287
Diameter = 4
Density = 0.106

of long diameter create new edges shortening the distance, and then decreasing the diameter and sounding the bandwagon effect. While the total number of edges, average degree and density rise monotonically, the diameter displays a non-linear behaviour, showing up an Granovetter (1978) threshold. On passing the threshold point, after Λ = 4, the bandwagon effect intensifies, and reach the maximum when all consumers are connected to each other. Considering a population of $n = 100$ consumers, as we assume in the simulations, the limit of density where each consumer is connected to all others is 1.0 or 100% which means the same that the total number of undirected edges given by $n(n - 1) = 9900$ or 99 edges for each node. This limit condition at maximum awareness, where everyone sees, hear or is inside the radius of everyone, is what we could name the Walrasian world network, a very special case of many others less omnipresent but more interesting world with limited relationship between agents. Remember that each class of consumer has a different propensity to follow

23

the bandwagon (see parameter $\beta$ in table 1), what means that some of then can be reluctant to go on board and assimilate new innovations. The bandwagon effect is restricted by $\beta$ and the distribution of the population. The Walrasian tatonnement refers to awareness radius only.

## 5.3. *Average results from different scenarios around the baseline*

Moving forward, the next intriguing question is what kind of the economic world, in terms of innovation, diffusion, saturation, price, quality and concentration emerge from each network topology and from different population composition? To answer that in this section we present the simulation results of the model in five different scenarios in which different values for two key variables are assumed: consumers awareness radius ($\Lambda = \{1, 1.5, 2, 2.5, 3\}$) and the percentage of techies amongst the population (techies $= \{3\%, 6\%, 9\%, 12\%, 15\%\}$). In order to ensure robustness, 100 simulation runs are performed for each scenario, for a total of 1000 runs over a time period of $t = 2000$. Results are presented for the most important variables in the model.

It will be shown that the impact of awareness radius and the percentage of techies in the proposed economy are quite similar, except for some few important peculiarities. However, it is fundamental to stress the difference between the logic and meaning associated with these two variables.

In this model, the awareness of an individual is a direct measure of rationality. If awareness radius is set to the maximum possible ($\Gamma/2$), that means consumers can perceive the consumption behaviour of every single agent in the economy at any point in time, which is rather unrealistic. But note though, that even so, it would not be possible to assume perfect rationality since the agents in this model cannot predict the future and foresee the decisions of consumers and firms alike. What can be safely argued is that awareness is a variable that probably does not change much from market to market. Of course, individuals have different interests and thus pay attention to different patterns of fashion but rationality is not expected to change that much and is probably not expected to be perfect.

The percentage of techies, on the other hand, simply refers to the number of people in the economy that are willing to buy new products as soon as they are released and thus it is a variable strongly related to the degree of the technology of the given market. It would be at least odd to watch people lining up early in the morning in front of the supermarket just to buy the new type of frozen lasagne a certain brand announced.

### 5.3.1. *Varying Awareness*

Changing the value of awareness radius has clear impacts on all variables of interest in the model. Let's start with the impact of awareness radius on market concentration (figure 6a below) . This relationship is not straightforward because of the endogenous nature of the model. A higher awareness not only leaves consumers more susceptible to adopt new products but also to pursue better prices. That means consumers will be changing suppliers more frequently which can be good for competition[43]. On the other hand, the higher perception of the consumer about the new products on the market may lead to a fast concentration of the market in favour of the innovating firm.

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[43]markup adjustments will be more frequent raising price competition but not necessarily market competition because firms that fail to reduce their costs through process innovation will be punished harder.

Additionally, since awareness affects saturation speed and maximum saturation before product innovation, it may leave a smaller time gap for firms to enter the market and start investing in innovation, providing some advantage to incumbent firms and even more to the firm that manages to be the first to create a new best-quality product. This also pressures firms to invest a bigger part of their profits on R&D, driving weaker firms out of the market. Results then suggest that the awareness values that generates more market competition is 1.5 and 2, while it is interesting to observe that an awareness value too low (1) generate a higher market concentration (HHI).
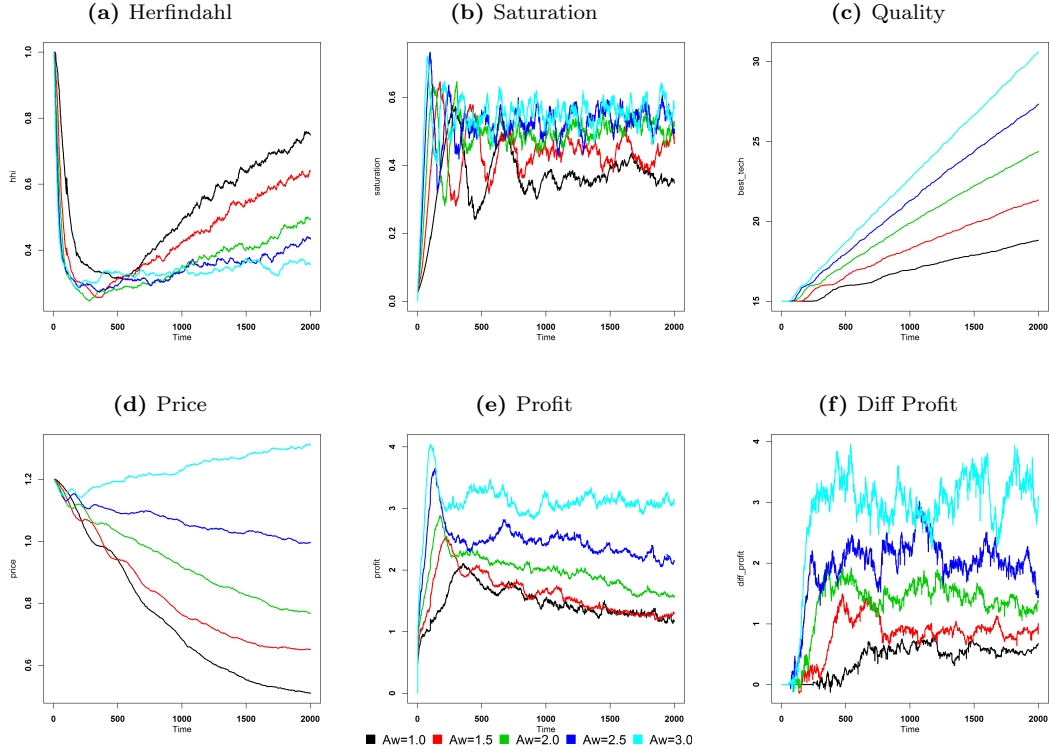
The impact of awareness on market saturation (figure 6b) is a bit tricky to observe since in every simulation the exact time of innovations arrival varies. Nevertheless, we can still observe that higher values of awareness yield saturation curves of higher frequency and relatively higher spikes, indicating that awareness positively affects saturation speed and the maximum saturation achieved just before a new product innovation arrives.

With a bigger awareness radius $\Lambda$, or rationality, consumers are able to observe more individuals and therefore the probability to be influenced rises. Thus, a new product is diffused faster in the market raising the speed of saturation and consequently yielding more innovations along the observed period of time. The higher the awareness the more frequent is the arrival of product innovations led by faster market saturation. Thus, the economy reaches higher product qualities (figure 6c). It can then leave the enticement that growth and development might also be higher, taking into consideration the important fact that there are no restrictions on technology, nor consumer budget restrictions in our model.

However, firms prices (figure 6d) are also positively influenced by awareness, specially in the high awareness cases, which means that as quality rises, price follows, leaving the question of what happens to the value of the product when two forces are acting at the same time increasing price by quality (product) innovation and decreasing price by productivity (process) innovation. Given the fact that our model does not have an explicit consumer budget restriction, these forces are working without restriction. However, this is not damaging to the functioning of the model. Firms profits (figure 6e) and product innovation profitability (figure 6f) also grow with awareness mainly because of the higher unbounded prices and costs, but also with the influence of the combination between market concentration and the speed of saturation.

The effects are also exponential time-wise. Until an awareness value of 2, prices and profits decline when awareness is equal or lower than 2, seem to hold constant through time in the 2.5 case and after this threshold, the proportional increments in cost due to a bigger presence of product innovations unbalances the market and leads to exponential growth of variables. This is a remarkable result of innovation that remains unexplained. Through the market mechanism revealed by this model, it is possible to demonstrate how the interacting dynamic of innovation, pricing and consumer behaviour generate macroeconomic dynamic patterns.

**Figure 6.:** The Impact of Different Awareness Radius on the Main Variables of the Model (Average of 100 random seed)



(a) Herfindahl

(b) Saturation

(c) Quality

(d) Price

(e) Profit

(f) Diff Profit

■ Aw=1.0 ■ Aw=1.5 ■ Aw=2.0 ■ Aw=2.5 ■ Aw=3.0

**Notes:** The figure shows the average trajectory computed on 100 simulations with different pseudo-random seed along 2000 time-step.

### 5.3.2. Varying Techies

The effects of varying the percentage of techies in the market are similar to those obtained changing awareness but with a few important peculiarities.

First, market concentration (figure 7a) is affected by the number of techies in a much more direct positive way so that a society filled with crazy for technology consumers might generate an oligopolistic or monopolist characteristic to its high-tech industries. The effect of techies over HHI is much more clear than its awareness counterpart because changing the composition of the population does not affect the ability of consumers to perceive better prices. In fact, it acts against it by raising the number of people who immediately buy the new product without any regard for its price.

About saturation, although it is observable that a higher techies percentage in the market also accelerates the speed of diffusion, it is not possible to infer that it raises maximum saturation (figure 7b) achieved during each cycle. This is expected, since, unlike awareness shifts, differences in the number of techies do not affect the acceleration[44] of innovation diffusion, only its speed.

The evolution of product quality (figure 7c) over time yielded the expected results although the difference between different techies scenarios is smaller than in the awareness case. The higher the number of techies in the market, faster is the market saturation, leading firms to heavily invest in product innovation.

Results on prices (figure 7d), profits (figure 7e) and product profitability (figure 7f)

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[44]Acceleration can be viewed as the second derivative of the life cycle (saturation) function.

also follow the same pattern as the awareness analysis, except this time they reach an even higher maximum value about the end of the simulation. The once again present explosive tendency may indicate that raising the percentage of techies above a given threshold may also cause a certain degree of disequilibrium in the model.

**Figure 7.:** The Impact of Different Number of Techies on the Main Variables of the Model (Average of 100 random seed)
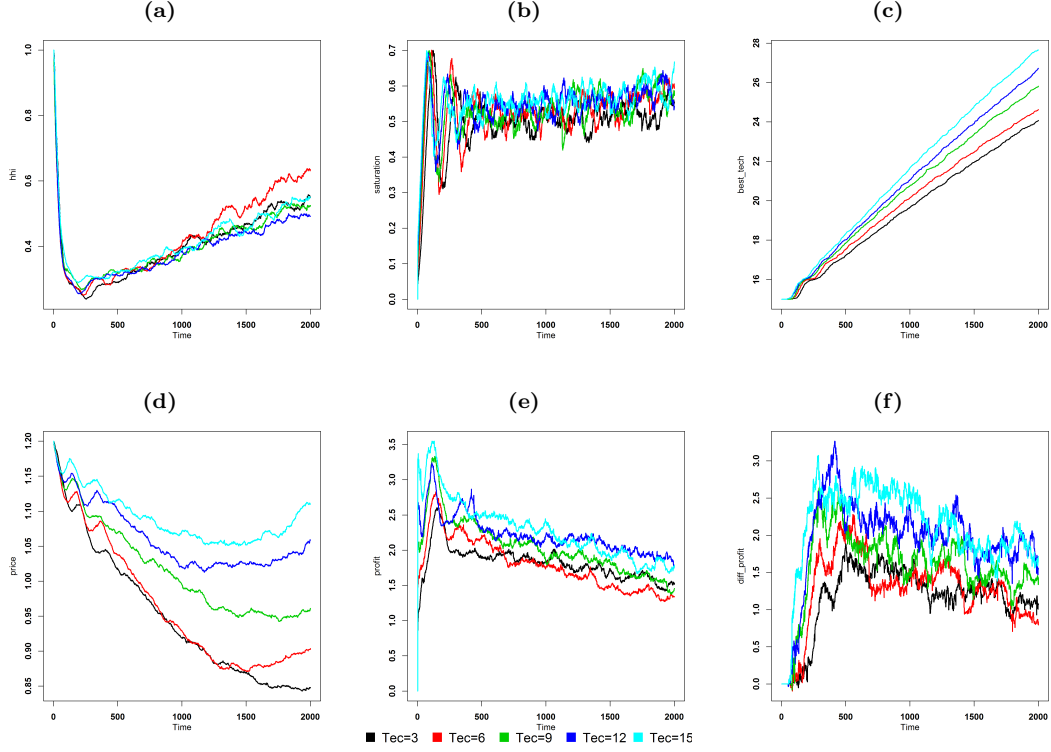


**Notes:** The figure shows the average trajectory computed on 100 simulations with different pseudo-random seed along 2000 time-step.

### 5.4.   *Wide and Smoothed Simulations for a range of Awareness and Populations*

There are still two important unanswered problems to be solved: how the amount of product and process innovations changes with the awareness radius and the distribution of the population and how quality is related to price in those cases. To be able to answer these questions, instead of analysing the evolution of variables over time, we perform a series of simulations to capture the final values of total product and process innovation, as well as the prices and quality. The results are shown in the scatter plots (8) and (9).

In the first case, the value of awareness ranges from 1 to 10 with a 0.1 variation. In the second, population distribution comprises all possible values of techies, visionaries, pragmatists, conservatives and skeptics in a range of 0% to 50% with a variation of 5 as long as the total population of consumers (newcomers in the simulation model) remains constant at 100[45]. For example, the first population composition is {0, 0, 0,

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[45]Which is the same population used in previous simulations.

50, 50} for each consumer types in order of bandwagon preference, which reflect an unyielding and skeptic society. The last one is {50, 50, 0, 0, 0} which correspond to a "loved for technology" society. In the first case, we are faced with a very *conservative* and *skeptic* society and in the last combination face with high *techies* and *visionarie* one.

Figure (8) show the results for the awareness range case. Each point corresponds to a different awareness value and curves are drawn to better visualize the emerging patterns. Confidence intervals for the fitness of the curve are presented around it. Population distribution is fixed and identical to the baseline, {3,12,35,35,15} respectively.

It is possible to draw interesting insights from both graphics. In figure (8), the value of awareness seems to initially have a positive effect on both process and product innovation, with the important fact that the ratio between process and product innovations seems to raise initially, which suggests that firms prefer process innovation when consumers have poor awareness radius mostly because the speed of diffusion is two slow and saturation takes time to go through the threshold of 50%[46]. But as awareness rises and diffusion speeds up, not only product innovation growth starts to slow down generating a concave curve, but process innovation starts to decrease, maintaining the average amount of total innovations almost constant.

This happens mainly because although the higher profits incurred with higher awareness radius fosters innovation of a single firm, the rising market concentration diminishes the number of firms in the market generating less total innovations.

The product quality curve follows a logistic shape, growing exponentially at lower levels of awareness and then slowing down at the higher radius. The price curve, on the other hand, falls at the beginning before adopting the same logistic shape as quality, but with a slightly bigger inclination that generates the catching-up of price and quality at higher levels of awareness. This pattern between the two variables generates a single point where the relationship between the quality a consumer gets for the price he pays for a product is maximum and happens when $\Lambda = 1.2$. Not surprisingly that is very close to the point where process innovation surpasses product innovation the most.

**Figure 8.:** The Impact of Awareness Range on Innovation, Product Quality and Prices



**(a)** Innovation by Awareness      **(b)** Quality and Price by Awareness

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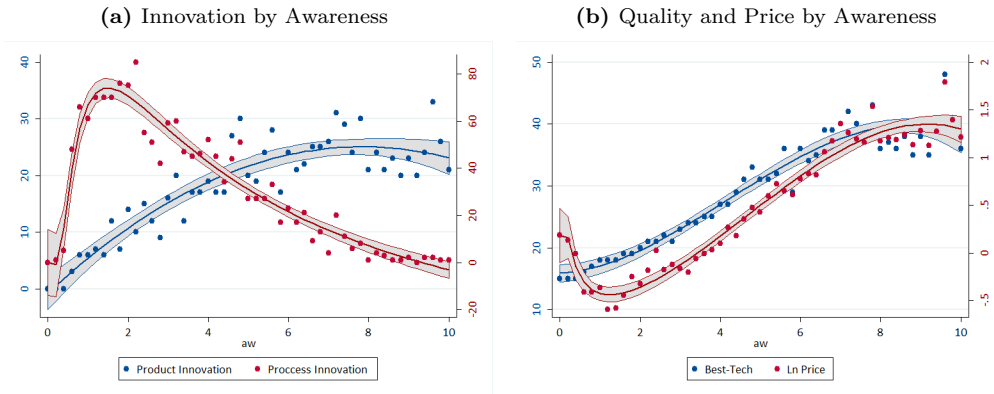
[46]As the reader can recall 50% is the threshold to shift investment from process to product innovation by the firm.

**Figure 9.:** The Impact of Population Distribution Range on Innovation, Product Quality and Price

**(a)** Innovation by Population          **(b)** Quality and Price by Population
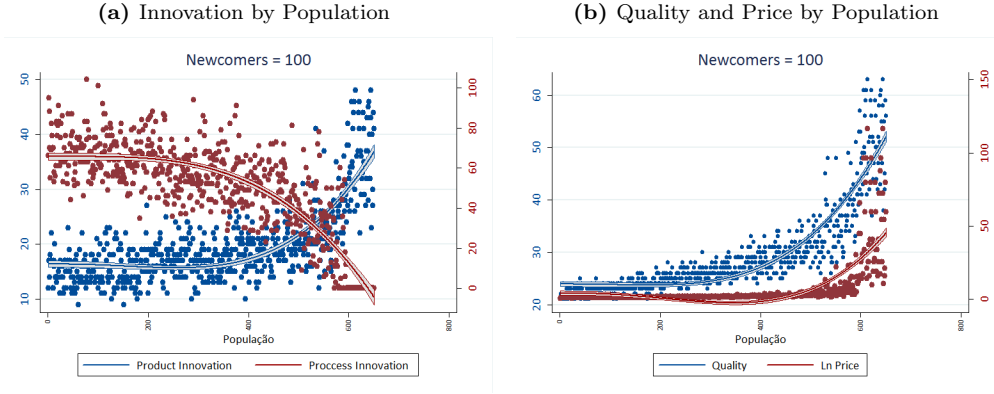


**Notes:** Figure 8 and 9 shows the state of selected variables at the end of simulation at time 1000. Each point in the scatter plot represents one simulation with a different parameters combination. In the case of figure 8, the *awareness* Λ is simulated in the interval [0,10] with smoothed increasing of 0.1, giving rise 100 simulations. In the case of figure 9, the population of consumer species {tec, vis, pra, con, ske} change from {0,0,0,50,50} through {20,20,20,20,20} to {50,50,0,0,0} giving rise to 651 valid combination. A valid combination is that the population sums up 100.

Figure (9) shows the results for the population distribution range with awareness fixed at 2.

Contrary to the awareness range case, product and process innovation curves start distant from each other with the firms heavily favouring process innovation through a big range of population distributions. However, product innovation curve has a positive exponential growth while process innovation one a negative exponential resulting in a fast catching up approaching the points corresponding to more technology friendly populations. The final result is similar to the previous case, total innovation remains basically constant through the observations and product innovation end up being higher than process innovation. It is the second derivative of the curves that change. The ones in awareness ranges case being negative and the present ones positive.

The acceleration of quality and prices curves are also different and exponential, with the evolution of quality being a little more gradual as the population becomes more tech-friendly and the evolution of prices suffering a fast explosion towards the end of simulations. In the range of observations described the ratio quality/price is always positive and growing.

## 6. Concluding Remarks

These are remarkable emergent dynamics that help to explain an important transition in the societies not sufficiently explained by the growth, innovation and quality literature. Technological change is a complex process where firms and consumer interact deeply, shaping the way that economic system evolute and agents behave. The technological civilization that is emerging in the course of last 200 or more year has experienced recently a huge acceleration impulsed by information technology which makes the social world, companies, institutions and people, more connected expanding their awareness radius and the behaviour of citizens with respect to the way they assimilate new technologies. This is not a new problem for observers of the history of societies. In 1970 Alvin Toffler wrote in his famous book "Future Shock" about the challenge faced by an innovative society: "Such changes in the ratio between old and

new have, as we shall show, *an electric impact on the habits, beliefs, and self-image of millions*. Never in previous history has this ratio been transformed so radically in so brief a flick of time", (Toffler 1970, p. 21, italic added). The *electric impact on the habits and beliefs* can be translated to our model as changing the awareness and consumer profile. If one is inclined to a more Panglossian view of the social evolution, accelerating the cycles of smarts innovation and their macroeconomic effects could prevent humanity from potential environmental catastrophe. This model could be easily transposed to this kind of discussion, with some emend of course.

In a more strict angle, the agent-based model depicted here has the goal to formalize and simulate the effects of bandwagon behaviour on the firm's decision between process and product innovation and on the macroeconomic dynamic emerging from this complex system. The chosen strategy was to test different values of consumer awareness - which in this model are closely related to the degree of agents rationality - and percentage of techies in the market, a type of consumer that immediately buys new products at the exact time they are released and thus are considered the initial seeds for innovation diffusion through a products life cycle.

Since the paper does not cover empirical confirmation for the model, results should be viewed with precaution and considered mostly theoretical. The theory depicted here can be useful for support future empirical studies, and the method developed and named SLWN useful to address many aspects of innovation and consumer theory in an interactive and more realistic framework. We present three types of results that intends to show different aspects of the model such as the mechanisms inside the model, and the evolution of variables through time and between different ranges of awareness and population distribution.

The percentage of techies seems to have a positive effect on the ratio between product and process innovation i.e. firms tend to value more product innovation over process innovation when the population is composed of more tech-friendly consumers with higher bandwagon coefficients. We also find that increasing population of techies has a positive effect on the speed of saturation, market concentration, product quality, innovation profitability, profits and price charged by the firms.

As for the awareness ratio of consumers, results suggest that the ratio between process and product innovation have two distinct phases. First, total innovation increases as well as the ratio between process and product innovations. From awareness radius 1.2 on, however, the total amount of innovation becomes stable and the ratio begins to smoothly decrease until product innovation is finally preferred.

Altogether all factors mentioned accelerating the transforming innovation and quality.

There are several ways in which this model could be upgraded in future research. A formal process of production decision could be incorporated, as well as labour market and consumer income inequality. The assumption of homogeneous awareness among consumers and through time could be relaxed to capture the idea that some consumers are better informed than others and also that consumer preferences towards technology changes with time (as the individual gets older for example).

The dynamics depicted in the present model helps to highlight essential connections between consumer behaviour by demand side and innovation by the supply side of an economy in a way that has remained slightly inaccessible in the literature. This complex dynamic is essential to understand the evolution of societies with more or less innovative aptitude. The frequency and type of innovation depend on the awareness and the heterogeneous preferences of the population. Although firms innovate to maximize profits, they also do it to adapt to the characteristics of the market demand,

in order to grow and survive. This could have an important impact on worldwide development. For example, if each new product embodies a better quality, and this better quality in the eyes of consumers means something such as an environmental friendly trait, the pattern of innovations could be shifted to ecologically sustainable growth more quickly.

The Small Live World Network developed in this paper provides an important contribution to social network literature and could also allow for several complements and different applications in the fields of economics and social science, such as search and matching systems, firms and stores location decision, marketing strategies and others.

## 7. References

### References

Acemoglu, D., A. Ozdaglar, and E. Yildiz. 2011. "Diffusion of Innovations in Social Networks." *MIT Working Papers* .

Adner, R., and D. Levinthal. 2001. "Demand Heterogeneity and Technology Evolution: Implications for Product and Process Innovation." *Management Science* 47 (5): 611–628.

Aghion, P., N. Bloom, R. Blundel, R. Griffith, and P. Howitt. 2002. "Competition and Innovation: An Inverted U Relationship." *NBER Working Papers* 9269.

Aghion, P., R. Blundel, R. Griffith, P. Howitt, and S. Plantl. 2006. "The Effects of Entry on Incumbent Innovation and Productivity,." *NBER Working Papers* 12027.

Aghion, Phillip, and Peter Howitt. 1992. "A Model of Growth Through Creative Destruction." *Econometrica* 60 (2): 137–147.

Aoki, M., and H. Yoshikawa. 2002. "Demand saturation-creation and economic growth." *Journal of Economic Behaviour and Organization* 48: 127–154.

Arndt, J. 1967. "Role of product-related conversations in the diffusion of a new product." *Journal of Marketing Research* 4 (3): 291–295.

Barro, Robert, and Xavier Sala-i Martin. 2004. *Economic Growth*. 2nd ed. EUA: MIT Press.

Bass, F. M. 1969. "A new product growth model for consumer durables." *Management Science* 15 (1): 215−227.

Bemmaor, A. C. 1994. *Modeling the diffusion of new durable goods: Word-of-mouth effect versus consumer heterogeneity*, 201−223. Boston: Kluwer.

Benhabib, Jess, Alberto Bisin, and Matthew O. Jackson. 2010a. *Handbook of Social Network - Vol 1B*. Amsterdan, The Netherlands: North-Holland.

Benhabib, Jess, Alberto Bisin, and Matthew O. Jackson. 2010b. *Handbook of Social Network-Vol 1A*. Amsterdan, The Netherlands: North-Holland.

Blume, L. 1993. "The Statistical Mechanics of Strategic Interaction." *Games and Economic Behaviour* 4: 387–424.

Bonanno, G, and B. Harworth. 1998. "Intensity of competition and the choice between product and process innovation." *International Journal of Industrial Organization* 16 (4): 495–510.

Brown, Jaqueline Johnson, and Peter H. Reingen. 1987. "Social ties and word-of-mouth referral behavior." *Journal of Consumer Research* 14 (3): 350–362.

Caliari, Thiago, Ricardo M. Ruiz, and Marco Valente. 2017. "Heterogeneity of demand and product innovation." *Estudos Econômicos* 47 (1): 5–37.

Chang, M. 2009. "Entry, Exit, and the Endogenous Market Structure in Technologically Turbulent Industries." *Cleveland State University Working Papers* .

Chugh, D., and M. H. Bazerman. 2007. "Bounded Awareness: what you fail to see cant hurt

you." *Mind & Society* 6 (1): 1–18.

Cowan, Robin, Willian Cowan, and G. M. Peter Swann. 2004. "Waves in comsumption with interdependence among consumers." *Canadian Journal of Economics* 37 (1): 149–177.

Cowan, Robin, Willian Cowan, and Peter Swann. 1997. "A model of demand with interactions among consumers." *International Journal of Industrial Organization* 15: 711–732.

Czepiel, J. A. 1974. "Word of Mouth Process in the Diffusion Process of a Major Technological Innovation." *Journal of Marketing Research* 11 (2): 172–180.

Dias, D., C. Marques, F. Martins, and J. Silva. 2011. "Why Are Some Prices Stickier Than Others?" *European Central Bank Working Paper* 1306.

Dichter, E. 1966. "How word-of-mouth advertising works." *Harvard Business Review* 44 (6): 147–166.

Dixit, A. 1989. "Entry and Exit Decisions under Uncertainty." *Journal of Political Economy* 97 (3): 620–638.

Ellison, Glenn. 1993. "Learning, Social Interaction, and Coordination." *Econometrica* 61: 1047–1071.

Ellison, Glenn, and Drew Fudenberg. 1995. "Word-of-mouth communications and social learning." *Quarterly Journal of Economics* 110 (1): 93–125.

Erdös, Paul. 1947. "Some remarks on the theory of graphs." *Bulletin of American Mathematical Society* 53: 21–24.

Erdös, Paul, and A. Rényi. 1959. "On random graphs." *Publications of Mathematicae* 6: 290–297.

Erdös, Paul, and A. Rényi. 1961. "On the evolution of random graphs." *Publications of the Mathematical Institute of the Hungarian Academy os Sciences* 5: 343–347.

Gaertner, W. 1974. "A Dynamic Model of Interdependent Consumer Behavior." *Zeitschrift für Nationalökonomie* 34: 327–344.

Giovinazzo, Viviana Di, and Ahmad Naimzada. 2015. "A model of fashion: Endogenous preferences in social interaction." *Economic Modelling* 47: 12–17. http://www.sciencedirect.com/science/article/pii/S026499931400501X.

Goldenberg, Jacob, Barak Libai, and Eitan Muller. 2010. "The chilling effects of network externalities." *International Journal of Research in Marketing* 27 (1): 4–15. http://dx.doi.org/10.1016/j.ijresmar.2009.06.006.

Granovetter, Mark S. 1973. "The Strength of Weak Ties." *The American Journal of Sociology* 78 (6): 1360–1380.

Granovetter, Mark S. 1978. "Threshold Models of Collective Behabiour." *The American Journal of Sociology* 83 (6): 1420–1443.

Grossman, G. M., and E. Helpman. 1991a. *Innovation and Growth in the Global Economy.* Cambridge, MA: MIT Press.

Grossman, G. M., and E. Helpman. 1991b. "Quality ladders in the theory of economic growth." *Review of Economic Studies* 58: 43–61.

Higuchi, T., and M. Trout. 2008. *Life Cycle Management in Supply Chains.* Hersheys, NY: IGI Publishing.

Keney, M., M. Lawless, and A. Murphy. 2010. "How do Firms Set Prices? Evidence From Ireland." *Central Bank and Financial Services Authority of Ireland Research Papers* 7RT10.

Kiesling, Elmar, Markus Günther, Christian Stummer, and Lea Wakolbinger. 2012. "Agent-based simulation of innovation diffusion: a review." *Central European Journal of Operations Research* 20 (2): 183–230.

Leibenstein, H. 1950. "Bandwagon, Snob, and Veblen Effects in the Theory of Consumers' Demand." *The Quarterly Journal of Economics* 64 (2): 183–207.

Lippert, Stephen, and Giancarlo Spagnolo. 2011. "Networks of relations and Word-of-mouth Communication." *Games and Economic Behaviour* 72: 202–217.

Marshall, Alfred. 1890. *The Principles of Economics*. 8th ed. London: Macmillan. Print edition 1949.

McMeekin, A., K. Green, and V. Walsh. 2002. *Innovation by Demand*. Manchester, UK: Manchester University Press.

Metcalfe, J. Stanley. 2001. "Consumption, preferences, and the evolutionary agenda." *Evolutionary Economics* 11 (1): 37–58. https://doi.org/10.1007/PL00003855.

Milgram, Stanley. 1967. "The Small World Problem." *Psychology Today* 2: 60–67.

Moore, G. 2005. *Dealing with Darwing*. New York: Portfolio.

Morgenstern, Oskar. 1948. "Demand Theory Reconsidered." *The Quarterly Journal of Economics* 62 (2): 165–201.

Mukherjee, Prithwiraj. 2014. "How chilling are network externalities? The role of network structure." *International Journal of Research in Marketing* 31 (4): 452–456. http://dx.doi.org/10.1016/j.ijresmar.2014.09.002.

Pasinetti, Luigi L. 1981. *Structural change and economic growth*. Cambridge: Cambridge University Press.

Pollak, Robert A. 1976. "Interdependent Preferences." *The American Economic Review* 66 (3): 309–320.

Pollak, Robert A. 1978. "Endogenous Tastes in Demand and Welfare Analysis." *The American Economic Review* 68 (2): 374–379. http://www.jstor.org/stable/1816724.

Pool, Ithiel de Sola, and Manfred Kochen. 1978. "Contacts and Influence." *Social Networks* 1: 1–48.

Porter, Michael. 1999. "Clusters e Competitividade." *Revista Management* July.

Rae, John. 1905. *The Sociological Theory of Capital*. United States: The MacMillan Co.

Reinstaller, Andreas, and Bulat Sanditov. 2005. "Social structure and consumption: on the diffusion of consumer good innovation." *Journal of Evolutionary Economics* 15 (5): 505–531.

Rogers, E. M. 1995. *Diffusion of Innovations*. 4th ed. New York: The Free Press.

Rosenkopf, L., and E. Abrahamson. 1999. "Modeling Reputational and Informational Influences in Threshold Models of Bandwagon Innovation Diffusion." *Computational & Mathematical Organization Theory* 5 (5): 361–384.

Rosenkrantz, S. 2005. *Simultaneous choice of process and product innovation*. Wissenschaftszentrum, Berlin: Mimeo.

Ryan, Bryce, and Neal C. Gross. 1943. "The diffusion of hynrid seed corn in two Iowa communities." *Rural Sociology* 8 (1): 15–24.

Saviotti, P. P., and A. Pyka. 2012. "On the Co-evolution of Innovation and Demand: Some Policy Implications." *Revue de l'OFCE* 124 (5): 347–388.

Schumpeter, Joseph A. 1934. *A Theory of Economic Development*. Cambridge, Massachusetts: Harvard University Press.

Smith, Adam. 1776. *An Inquiry into the Nature and Causes of the Wealth of Nations*. New York: Modern Library. Printed in 1936".

Toffler, Alvin. 1970. *Future Shock*. New York, USA: Bantam Books.

Travers, J., and S. Milgram. 1969. "An experimental study of the small world problem." *Sociometry* 32: 425–443.

Valente, Marco. 2008. "Pseudo-NK - an enhanced model of complexity." *LEM Working Papers* 2008/11.

Valente, Marco. 2012. "Evolutionary demand: a model for boundedly rational consumers." *Journal of Evolutionary Economics* 22: 1029–1080.

Valente, Thomas W. 1996. "Social Network Thresholds in the Diffusion of Innovations." *Social Networks* 18: 69–89.

Veblen, Thorstein. 1899. *The Theory of the Leisure Class: An Economic Study in the Evolution*

*of Institutions*. United States: Macmillan.

Wasserman, Stanley, and Katherine Faust. 1994. *Social Networks Analysis*. Cambridge-UK: Cambridge Press.

Watts, Duncan J. 1999. "Networks, Dynamics, and the Small-World Phenomenon." *American Journal of Sociology* 105 (2): 493–527.

Watts, Duncan J., and Steven H. Strogatz. 1998. "Collective dynamics of "small-world" networks." *Nature* 393 (6684): 440–442.

Young, P. 2002. "The Diffusion of Innovations in Social Networks." *Santa Fe Institute Working Papers* 13–54.

Zbaracki, M., M. Ritson, D. Levy, S. Dutta, and M. Bergen. 2003. "Managerial and customer costs of price adjustment: direct evidence from industrial markets." *The Review of Economics and Statistics* 86: 514–533.

# Appendix A. Symbols and Values

**Table A1.:** Model Variables

| Symbol | Meaning | Equation |
|--------|---------|----------|
| $\Gamma$ | Economic space as a cartesian plane | * |
| $\Omega$ | Set of all consumers in the model | 1 |
| $\Phi_z$ | Subset of consumers who adopted a given product $z$ | 1 |
| $\Omega_i$ | Subset of individuals whithin the awareness radius of consumer $i$ | 1 |
| $\theta_i^{p,q}$ | Probability of consumer buy a product by price or quality | 1 |
| $\beta$ | Stochastic bandwagon threshold | 1 |
| $e_{i,}$ | Effort of consumer $i$ to buy a given product | 2 |
| $\rho_{i,\chi}$ | Price paid by consumer $i$ to a firm $\chi$ for a given product | 2 |
| $q_{z,t}$ | Quality of product $z$ | * |
| $d_{i,\chi}$ | Distance between the consumer $i$ and his supplier $\chi$ | 2 |
| $\chi_i$ | Firm chosen as supplier by consumer $i$ | 3 |
| $\Xi_i$ | Subset of firms that can potentially influence consumer $i$ | * |
| $p_x$ | Price charged by firm $x$ | 6 |
| $\Theta_x$ | Percentage of profits a firm secures for R&D | 8 |
| $S_x$ | Market Saturation of firm $x$ | 8 |
| $HHI$ | Herfindahl Hirschman Index | 8 |
| $ms_x$ | Market-share of a given firm | 10 |
| $I_x^{R\&D}$ | Investment of firm $x$ in R&D | 11 |
| $\pi_x$ | Profit of firm $x$ | 12 |
| $D_x$ | Demand of firm $x$ | 12 |
| $c_x$ | Cost of firm $x$ | 12 |
| $I_x^{PD}$ | Investment in product innovation of firm $x$ | 13 |
| $I_x^{PC}$ | Investment in process innovation of firm $x$ | 13 |
| $\phi$ | Probability of arrival of an innovation of a given type | 14 |
| $\Psi_x$ | Set of products produced by firm $x$ | 15 |
| $\mu_x$ | markup of firm $x$ | 17 |

**Table A2.:** Model Constants

| Symbol | Meaning | Equation | Values |
|--------|---------|----------|--------|
| $k_1$ | Weight of each type of influence on consumers | 1 | 0.5 |
| $k_2$ | Weight of firm's distance on a consumer effort | 2 | 0.03 |
| $k_3$ | Controls how easy consumer decides to change products | 5 | 0.05 |
| $k_4$ | Controls the impact of saturation on $\Theta_x$ | 8 | 0.2 |
| $k_5$ | Controls the impact of saturation on $\Theta_x$ | 8 | 0.2 |
| $k_6$ | Controls the impact of HHI on $\Theta_x$ | 8 | 0.2 |
| $k_7$ | Controls the impact of HHI on $\Theta_x$ | 8 | 0.2 |
| $k_8^{PD}$ | Controls the impact of $I_x^{PD}$ on $\phi$ | 14 | 0.01 |
| $k_8^{PC}$ | Controls the impact of $I_x^{PC}$ on $\phi$ | 14 | 0.08 |
| $k_9$ | Impact of $\mu$ on a firms price setting decision | 17 | 0.60 |
| $k_{10}$ | markup changing reaction | 18 | 0.05 |
| $\Delta_c^{PD}$ | Positive variation on cost caused by a product innovation | 15 | 0.20 |
| $\Delta_c^{PC}$ | Negative variation on cost caused by a process innovation | 16 | 0.044 |
| $r$ | Interest rate | 21 | 0.05 |
| $c_{z,x}(0)$ | Initial cost of firms | * | 1.0 |
| $\mu_x(0)$ | Initial markup of firms | * | 0.2 |
| $Q_Z(0)$ | Initial quality of the first product in the model | * | 1 |